THE
MATHEMATICAL GAZETTE.

EDITED BY
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WITH THE CO-OPERATION OF
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THE EDINBURGH MATHEMATICAL COLLOQUIUM.

THE first Mathematical Colloquium held in Edinburgh met during the first week of August, and proved a great success. It was organised by the office-bearers of the Edinburgh Mathematical Society, A. G. Burgess, M.A., F.R.S.E., and Peter Comrie, M.A., B.Sc., F.R.S.E., respectively the President and Secretary of that Society, being also President and Secretary of the Colloquium. The idea of holding such a colloquium was an outcome of Professor Whittaker's announcement that he purposed organising, as part of the Mathematical Honours curriculum in the University of Edinburgh, a mathematical laboratory for systematic numerical discussion of functions and methods of calculation. Several correspondents had expressed the hope that vacation courses in this line of study might be established; and it was decided to make a first experiment. It was resolved, however, not to limit the colloquium to a discussion of one branch of mathematics, but to enlarge its scope by the inclusion of two other domains of mathematical thought. The broad features of the programme we owe to Professor Whittaker; and its variety was such as to appeal to all types of mathematical mind.

By a curious chance the three courses of lectures provided were given respectively by an Englishman, an Irishman, and a Scotchman. Each day at ten o'clock Professor Conway of University College, Dublin, discoursed on the Theory of Relativity and the new Physical Ideas of Space and Time. At 11.30 Professor Whittaker explained practical Harmonic Analysis and Periodogram Analysis; and at two o'clock Dr. Sommerville of St. Andrews expounded the mysteries of non-Euclidean Geometry and the Foundations of Geometry.

Professor Conway and Dr. Sommerville lectured in the Mathematics Class-room of the University; but Professor Whittaker held his *séances* in the large basement hall, which has recently been fitted up as a mathematical laboratory. This, indeed, was the first occasion on which it had been used. Each student was provided with a specially designed desk, with a convenient book-rest fixed to the back, and with drawers and shelves for storing note-books, scribbling paper, graph paper, and books to aid calculation, such as Barlow's *Tables* and Crelle's *Rechentafeln*. At these desks learned professors, lecturers, teachers, and a few students, nearly eighty in all, totted up their columns of figures and drew their periodographs, and were quite elated when their totals came out right. Professor Whittaker chose for his working data the light

periods of two variable stars, the one to illustrate the periodogram method of discovering unknown periods, the other to illustrate the analysis into harmonic components of a given periodic variation. The theory of the Fourier analysis was incidentally given; and the last lecture finished with an account of Mäder's Harmonic Analyser.

This hour of practical work, combined with demonstrations involving only the familiar circular functions, gave the necessary balance to the weird imaginings of the other two courses. Without it to bring us back to the obvious world of apparent realities we should have been floundering hopelessly in the Absolute or in Minkowski's *Welt*. After we had been taught that velocities did not compound according to the parallelogram law, it was a positive delight to find that the Fourier series remained ordinarily additive; and with this in possession we had no great difficulty in apprehending the possibility of a space devoid of parallel lines.

One of our number, who hailed from Dundee, had been renewing his acquaintance the preceding evening with Gilbert and Sullivan's "Patience." When an exceptionally imaginary theorem was enunciated and *proved*, he turned round to his friend in the bench behind and whispered, "Yes, it *is* nonsense, but oh! such *precious* nonsense!"

Newtonian dynamics, we found, was only a first approximation to the dynamics of our visible universe; while the Euclidean space in which this universe was vulgarly believed to move and have its being was a crude assumption from an axiom of ignorance. Again our Dundee professor hit off the situation with the quatrain,

The classic phoenix, sprung from fire,
Fable no longer dare we scorn;
Euclid and Newton both expire—
Conway and Sommerville are born!

Sandwiched in between these physical and mathematical heresies came the hour of comparative mental rest and the hour of physical recuperation in the luncheon room, and we were saved from unbelief in the realities of life. We learned many things. We were told that even if the theory of relativity were not true it had taught us truths. The tendency of modern Physical Theory was in the direction of still further atomising the atom; yet it was necessary in geometry to have an assumption of continuity, so that all possible numbers might be brought into correspondence with an infinitude of points on a finite line. The dictum of the logician that we cannot define by means of a negation seemed to have no terror to the modern geometer with his glib talk of non-Euclidean, non-Pascalian, non-Desarguesian, and even non-Archimedean.

It was indeed a grand week. Each lecturer surpassed himself; and every one of the audience enjoyed himself or herself to the full. Professors and lecturers of world-wide reputation renewed their student days, and sat and took notes with humble zest. Professor Steggall, in moving a vote of thanks to the lecturers and to the organisers of the colloquium, hoped that this experience of taking notes for one brief week would make us feel more sympathetic with the unfortunate students who had to do the same for weeks and months. Under the leadership of the three lecturers we had been seeing visions and dreaming dreams; and he was sure we would all look back upon this colloquium with the keenest delight.

The three hours' hard mathematical thinking was followed every afternoon by suitable recreation. Mr. Burgess and Mr. Comrie arranged for golfing parties to the principal golf courses in the neighbourhood—Mortonhall, Barnton, and Baberton. On the Wednesday afternoon it was my privilege to lead a party through the beautiful grounds of the

new Scottish Zoological Park at Corstorphine. The lions, bears, jackals, monkeys, parrots, antelopes, and the antique-looking gnu were all duly admired or wondered at. A discussion arose as to whether a certain four-footed creature was a camel or a dromedary. The problem was finally solved by the formula, ln=constant, where l is the name-length and n the hump-frequency. A charming hour was spent taking tea on the lawn in front of Corstorphine House, from which was a splendid view of the Pentland Hills. Reminiscences of college days in Cambridge, Dublin, and Edinburgh formed the nucleus of a varied gossip, which had usually a certain mathematical flavour.

On the Thursday afternoon a large party were shown over the statistical department of the Register House by Dr. Dunlop, who demonstrated the striking mechanical methods by which groups of statistics could be sifted and arranged in any required combination.

On the Friday, after Dr. Sommerville's last lecture, the whole company were entertained to tea in the Reception Room (the Mathematical Library) by the President and Secretary; and thereafter visitations were made to the University Physical Laboratory under the leadership of Dr. Carse, and to the Heriot Hospital School, where Mr. Gentle and others of the staff showed what is probably the most complete school laboratory for science teaching to be found in Scotland.

In formally closing the colloquium, Mr. Burgess attributed the success of the gathering to the untiring labours of the Secretary, Mr. Comrie, to the splendid efficiency of the lecturers, and to the good fellowship which had existed. This was the first colloquium; but he hoped it would not be the last. Possibly next year it might be combined with the Napier Tercentenary Celebration. C. G. Knott.

THE TEACHING OF GEOMETRY AND TRIGONOMETRY.

(*Continued from* p. 146.)

IV. Areas.

Area of a Rectangle.—Consider a rectangle A of height h and base b (Fig. 18). By moving the rectangle so that its base slides along the x-axis through a distance b, it takes up the second position A. Thus, on a length $2b$ of the x-axis, commencing at O, we have a rectangle of height h and area $2A$, and so on.

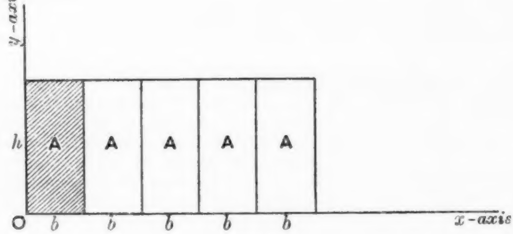


Fig. 18.

On each $\frac{1}{10}b$ we have $\frac{1}{10}A$, on each $\frac{1}{100}b$ we have $\frac{1}{100}A$, and so on.

Thus, *If the base of a rectangle is multiplied by n without altering its height, then the area is also multiplied by n, where n is any number, not necessarily a whole number.*

A double application of this principle shows that if the base is multiplied by n and the height by m, then the area is multiplied by mn.

Hence, *If the base of a rectangle is b units of length and the height h units of length, then the area is bh units of area.*

It is not even necessary that the base and the height should be measured in terms of the same unit of length.

More briefly, *The area of a rectangle is the product of its base and height.*

It is easy to proceed to consider the areas of a parallelogram, a triangle, and a trapezium. At this stage also it is convenient to introduce geometrical illustrations of algebraic formulae.

Pythagoras' Theorem.—Here is another proof of Pythagoras' Theorem, involving only the areas of a square and of right-angled triangles. It was designed for a Civil Service Examination, and is reproduced by kind permission of the Controller of His Majesty's Stationery Office.

Let ABC be a right-angled triangle, right-angled at C, and let $b > a$ (Fig. 19).

Let ABC be rotated through a right angle round A into the position ADE. Then $ACFE$ is a square of side b and area b^2.

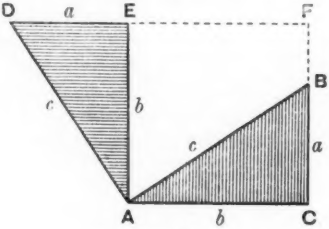


Fig. 19.

As the part ABC is taken away and put on again in the position ADE,

$$\text{area } ABFD = b^2.$$

But $$\text{area } BAD = \tfrac{1}{2}c^2$$

and $$\text{area } BFD = \tfrac{1}{2}(b+a)(b-a)\,;$$

$$\therefore\ b^2 = \tfrac{1}{2}(c^2 + b^2 - a^2),$$

whence $$a^2 + b^2 = c^2.$$

Distance between two Points in Rectangular Co-ordinates.—At this stage it is easy to consider the distance between two points whose rectangular co-ordinates are all positive. The principle may be applied to prove the

Converse of Pythagoras' Theorem.—Let ABC be a triangle in which it is known that $c^2 = a^2 + b^2$. Of the two angles A and B, one, at least, is acute. Supposing then that A is acute, take AC as x-axis and the y-axis at right angles through A (Fig. 20). Let (x, y) be the co-ordinates of B. Then the equation which expresses that c^2 is the sum of a^2 and b^2 is

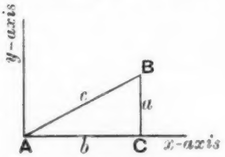


Fig. 20.

$$x^2 + y^2 = 2b^2 - 2bx + x^2 + y^2\,;$$

$$\therefore\ x = b\,;$$

$$\therefore\ ACB \text{ is a right angle.}$$

Of course another proof may be given later.

Objection will scarcely be made that this proof is too difficult for boys of twelve, but perhaps that it is too easy.

V. Symmetry.

The subject of symmetry may be introduced by the use of a folded sheet of paper and a pair of scissors in the kindergarten.

The Bisector of an Angle.—Let the straight line BOA bisect the angle P_1OP, and let OP_1 be taken equal to OP (Fig. 21).

By folding about AB it is seen that every point in AB is equidistant from P and P_1, and that PP_1 is bisected at right angles by BA.

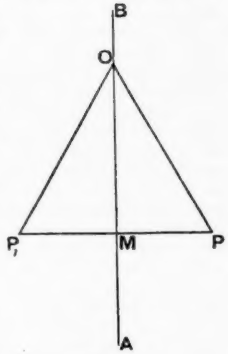


Fig. 21.

Perpendicular Bisector of a Line-Segment.—The particular case in which the angle P_1OP is an *angle of continuation* is of special importance. Thus, *Every point in the perpendicular bisector of a line-segment is equidistant from its extremities.*

The above principles may then be applied to bring out the important properties of isosceles triangles already independently established, and under this head much of the geometry of the circle may be taken.

VI. General Considerations.

It will be seen that this system of Geometry is not based upon the Congruence of Triangles, but upon *Motion of Rotation*, *Motion of Translation* and *Folding*. This I have consistently advocated for many years. I also venture to suggest that Pure Geometry, Co-ordinate Geometry, Trigonometry and Mensuration may be very profitably treated as one subject. I do not wish to say one word in disparagement of Euclid's monumental work. But, surely it is not a little remarkable that a system of Geometry invented over 2000 years ago for university students before the days of Algebra, Trigonometry or even Arithmetic, should still obtain in schools where the vast majority of boys leave at the early age of sixteen. We have at last got Geometry into the Algebra; can we not also have the Algebra in the Geometry?

There are three serious objections to such a course:

(i) The first is that it is too difficult. This objection is raised by those teachers who say that there should be no attempt to prove anything at school. No boy should be expected to do as much thinking as would give a headache to a caterpillar.

(ii) The second objection is that the course is too easy. This is raised by those teachers who hold that it does not matter what a boy learns at school so long as he does not like it.

(iii) The third objection is still more serious. Examining bodies have proclaimed it as one of the Laws of the Medes and Persians that "all proofs of geometrical theorems must be geometrical." So, under the pressure of these examinations, vast multitudes of boys leave school knowing a fair amount of arithmetic, a little algebra, and less geometry entirely of a one-sided kind which they do not know how to use. The science masters are crying out for an early introduction to Trigonometry. The geography masters want Trigonometry and Similar Triangles taught early. The only masters who have to be tied up are the mathematical masters. As a matter of fact, the methods of Pure Geometry and Analytical Geometry are complementary,—they dovetail into each other in a most beautiful way; and it is a shame that the restriction to which I have referred should be insisted upon. If we must have separating walls, let them all be built upon arches, so that there may be free intercourse at least upon the ground floor. Much more can be said in favour of introducing at a later stage certain restrictions

designed to test the ingenuity of university students. But in these schools, let us aim at developing in our pupils the power to exercise choice of method, and let us widen the range as much as is practicable.

Allow me to conclude by a quotation from a book recently published in America, entitled, *The Teaching of Geometry*, and written by David Eugene Smith:

"The efforts usually made to improve the spirit of Euclid are trivial. They ordinarily relate to some commonplace change of sequence, to some slight change in language, or to some narrow line of applications. Such attempts require no particular thought and yield no very noticeable result. But there is a possibility, remote though it may be at present, that a geometry will be developed that will be as serious as Euclid's, and as effective in the education of the thinking individual. If so, it seems probable that it will not be based upon the congruence of triangles, by which so many propositions of Euclid are proved, but upon certain postulates of motion. . . . If to the postulate of parallel translation we join the two postulates of rotation about an axis, leading to axial symmetry; and rotation about a point, leading to symmetry with respect to a centre, we have a group of three motions upon which it is possible to base an extensive and rigid geometry. It will be through some such effort as this, rather than through the weakening of the Euclid-Legendre style of geometry, that any improvement is likely to come. . . . At present the important work for teachers is to vitalize the geometry they have, . . . seeing to it that geometry is not reduced to mere froth, and recognizing the possibility of another geometry that may sometime replace it,—a geometry as rigid, as thought-compelling, as logical, and as truly educational."

W. J. Dobbs.

NOTES ON THE RADIX METHOD OF CALCULATING LOGARITHMS.

(*Continued from* p. 150.)

A somewhat obvious simplification of Briggs' logarithmic process was discovered and given as one of three methods by Robert Flower in a rare small quarto tract, *The Radix a new way of making Logarithms*, published in London by J. Beecroft in 1771. Several tables of radices are given, the largest extending from $r=1$ to 9 and n from 1 to 12 to twenty-three places.

Flower divides the given number, if necessary, by a power of ten and a single digit, so as to reduce the first figure to ·9, and then multiplies by a succession of radices until all the digits become nines. The complement of the sum of the logarithms of the radices to the logarithm of the divisor gives the required logarithm. In some cases it is more convenient to multiply than to divide by a digit, the logarithm of the digit is then added to those of the radices and the complement to 1 taken.

Thus, Ex. iii., Flower finds log 3·9784; divide by 4.

3·9784	(log 4) =	+ 0·6020 5999
0·9946	(log 1·005) =	− 21 6666
0·9995 73	(log 1·0004) =	− 1 7368
0·9999 7283	(log 1·00002717) =	− 869
		− 304
		− 4
		− 3
	log 3·9784 =	0·5997 0845

The figures in brackets are not given by Flower.

Again, to find log π, Ex. vii., divide by 4 and multiply the quotient up to 1, subtract the logarithms of the radices from log 4.

This method was rediscovered and published as new by Hearn, 1847; it is generally called after him.

Each radix in general consists of 1 followed by as many ciphers as there are nines already obtained, and the complement to nines of the next one, two, or three digits. Hence they can be determined by simple inspection, and the somewhat tedious divisions are replaced by easy multiplications. When half the digits are nines, the complements of the remainder to nines give the required radices directly, and may be multiplied by μ.

It is occasionally necessary, especially when two or three digit radices are used, 'to force' or add one to the complement. This is so easily done by simple addition that it is hardly worth while to give a formula. Suppose c is the complement to nines of a period which follows nines, and d, e, are the next periods, the latter of which may almost always be neglected, the period becomes nines if

$$d+\frac{e}{10^n}\geqq\frac{c(c+1)}{10^n}.$$

Thus to find the smallest value of d which will make the second period nines in the case of ·999 888,

$$c=111,\quad d=111\times112\times10^{-3}=012\,432,$$

$$0\cdot999\,888\,012\,432\times1\cdot000\,111=0\cdot999\,999\,000\,001.$$

The method seems to have been very rapidly forgotten, and is dismissed by Hutton in a few words without description. It was revived in a most inconvenient form in *Phil. Trans.*, 1806, by Manning, who used negative radices, in which r was always one, so that a very tedious series of simple subtractions was necessary.

A very full account of the history of the method, with valuable criticisms, is given by A. J. Ellis, *Proc. R.S.*, xxxi. 398 and xxxii. 377, 1881. He attributes most of the modifications which have been subsequently proposed to George Atwood, of "machine" fame, in *An Essay on the Arithmetic of Factors*, printed, possibly for private circulation, by T. Cadell, London, 1786. "Hence it appears that Atwood rediscovered Flower's method, but transformed it in the manner carried out ninety years later, 1876, by Hoppe, and not only anticipated Weddle's method, 1845, but showed the connection of the two methods as that of multiplying the reduced number up to 1 in the first case, and down to 1 in the second."

Atwood gives positive and negative natural radices only in a new form to thirteen decimal places. From (i) $\log(1\pm x)$ is equal to $\pm x$—a correction which is tabulated by Atwood,

$$\log(1\pm\cdot01)=\pm\cdot01-\cdot0^45\pm\cdot0^6\dot{3}-\cdot0^825\pm.$$

Hence the tabular positive correction to be subtracted from $+\cdot01$ is $-\cdot00004\,96691\,468$, and the tabular negative correction to be added to $-\cdot01$ is $-\cdot00005\,03358\,535$. These corrections may be dealt with separately, and their sum added or subtracted at the end of the operations.

The use of potential radices of the form $\left(1+\frac{1}{10^n}\right)^r$, where r varies from 1 to 9, has been advocated by Orchard, 1848, and Oliver Byrne, 1849, but Ellis remarks, "Although from a potential radix the logarithm of a number can be obtained with the same accuracy as from a numerical radix, yet the process is much longer with the former; and hence it appears that the real use of the potential is to calculate the numerical

radix." With much deference to Mr. Ellis' opinion, it seems very doubtful if it is not more simple to calculate the numerical radices directly.

There is also much difference of opinion as to the utility of negative radices. If both kinds of radices are given, some few figures may be saved, but with greatly increased risk of errors of sign. It is not very practical to obtain a number from its logarithm by negative radices, since straightforward working gives the reciprocal and not the number. Hence it is usual to tabulate the complements of the logarithms of the radices and to take the complement of the given logarithm.

It may be of interest to apply the logarithmic and anti-logarithmic processes to the prime 8291, the error in each case being in the eleventh figure.

8291 × 13	Comp. log 88605 66476 9
107783 × $1{\cdot}0\bar{7}$	3151 70514 5
10023819 × $1{\cdot}00\bar{2}$	86 94587 1
10003771362 × $1{\cdot}000\bar{3}$	13 03078 9
100007702305914 × $1{\cdot}0000\bar{7}$	3 04016 8
7017667 × $1{\cdot}00000\bar{7}$	30401 7
1000000017618 × μ	76 5
log 8291	·91860 69151 4

8291/9 =	log 9	·95424 25094 393
92122222222 × 1·08	log	03342 37554 869497
99492 × 1·005		216 60617 5650
9998946 × 1·0001		4 34272 7686
99999 458946		23497 676
Comp. 541054		03563 55942 879
log 8291		·91860 69151 5

− 91860 691514		99999 55778 5 × $1{\cdot}000\bar{8}$
− 08139 308486		7 99996 4
− 08042 190762	83	99991 55782 1 × $1{\cdot}00\bar{1}$
− 47 117724		99 99155 8
− 43 451177	$1{\cdot}00\bar{1}$	99891 56636 3 × 83
− 3 666547		8290·9 99999 8
− 3 474495	$1{\cdot}000\bar{8}$	
− 192052 × $1/\mu$		
1 − 442215 = 557785		

·91860 691515		1·00000 66069 0
91381 385238	82	8 00005 3
479 306277		1·00008 66074 3
432 137378	1·01	100 00866 1
47 168899		1·00108 66940 4
43 407748	1·001	1001 08669 4
3 761151		1·01109 75609 8 × 82
3 474217	1·00008	202 219 51219 6
286934 × $1/\mu$ = 660690		8088 780 48784
		8291·000 00003 6

On the whole, for general use, no improvement seems to have been made on the original method of Briggs as modified by Flower; but the work is

shortened by tables of radices to two or three digits, if multiplication tables are available.

The general neglect of such a simple and convenient method may have been due to the idea that Briggs' radix method required the use of large and expensive tables, which is far from being the case.

Flower's pamphlet is very scarce and very confusing. He possibly calculated the logarithms of the radices by the aid of a table of the successive square roots of ten, and speaks almost as though he considered the nine digits to be roots of ten. He also mixed up the difficulty he found in calculating the logarithms of the radices with the much more simple matter of using them when found.

Atwood's pamphlet is still more scarce; there are copies in the libraries of the Royal and Royal Astronomical Societies, but not in the British Museum; since it deals with natural logarithms only, it would be of little use to practical computers. It seems worthy of reprinting.

A copy of Flower's tract came into the hands of Leonelli, who published his *Supplément Logarithmique* at Bordeaux in 1802, and an edition was published at Dresden by Leonhardi in 1806. He gave tables of natural and common positive radices to twenty places, and also a table of common two-figure radices to fifteen places. Gray claimed as new a two-figure negative table, 1846, and a two-figure positive table, 1848. The *Supplément* was also so scarce as to be almost unknown until Hoüel reprinted the tables of radices in 1858 and 1866, and the whole work in 1876. Schrön also reprinted the positive radices to sixteen places in 1859.

Perhaps the most convenient and powerful tables of three-figure radices are those of Gray, 1876, to twenty-four places. The one-figure positive and negative tables of Thoman to twenty-seven places, Paris, 1867, seem to be out of print and difficult to procure second-hand.

SYDNEY LUPTON.

THE SIMPLE PENDULUM.

THE current elementary discussion of the Simple Pendulum is unsatisfactory, in that the problem is not reduced with sufficient directness to a case of S.H.M. It has to be artificially prefaced by consideration of a curvilinear motion in which the tangential resolute of the acceleration is proportional in magnitude to the arcual distance from a point of the path; and the discussion of this motion raises new points of difficulty altogether out of proportion to its significance in this connection. The net result is that the student's appreciation of this first of the important applications of S.H.M. to "small oscillations" is lost, in dismay at finding that, even after he has mastered the essential difficulties of the S.H.M. itself, the first good application of it brings yet another awkward hurdle. And the physicist or engineer may well make this another case for railing impatiently at the devices for "dodging the Calculus" by which elementary theory is so apt to be obscured. Nevertheless, the teacher of Mathematics knows how important it is to postpone such Calculus difficulties as, *e.g.*, to pave the way to the complete primitive of the S.H.M. differential equation and its uses (one of which gives the only clear-cut way of handling the ordinary discussion of the Simple Pendulum). But the postponement must not be obtained at the disproportionate cost of artificial complications which, for the sake of "elementary" treatment, cast a fog round the important features of the argument, without contributing anything of independent value to the student's store of knowledge.

The elementary treatment of S.H.M. is well worth preserving. The relation

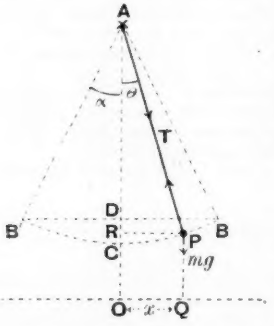
of S.H.M. to the uniform circular motion is of fundamental importance to a sound knowledge of S.H.M.; and the general gain from the elementary investigation of the S.H.M. equations is a better understanding of the fundamental dynamical method of resolution—from the opposite point of view to that which obtains in the other elementary case, viz. that of the (parabolic) motion of constant acceleration.

It is this principle of resolution that wants emphasising in the theory of the Simple Pendulum. The horizontal resolute-motion is what really interests us, in the motion of a pendulum; and consideration of it keeps a close analogy with the parent-theory of the S.H.M.: another case of a straight motion related as resolute to a circular motion.

(i) With an obvious notation (see diagram), if f_x denote the measure of the horizontal resolute of the acceleration, the equation of horizontal resolution is

$$m \, . \, f_x = -T \, . \sin\theta$$
$$= -T \, . \, \frac{x}{l}.$$

This is not an equation of S.H.M., since T will obviously vary during the motion; but for a *small* oscillation a first approximation may be obtained by neglecting squares and products of small quantities, i.e. *quantities which would have the zero value if the pendulum were at rest in its equilibrium-position AC*. Hence, since x and f_x are small quantities, as thus defined, and since T differs from $m \, . \, g$ (its equilibrium value) by a small quantity, it follows that the first approximation to the equation of horizontal resolution is the S.H.M. equation



$$f_x = -(g/l) \, . \, x.$$

Thus *the horizontal motion is approximately a* S.H.M., and the measure of its period $= 2 \, . \, \pi \, . \, \sqrt{l/g}$.

(ii) The discussion may be made complete by actually expressing T in terms of a geometrical variable, as follows. The figure explains itself.

Using the equation of normal resolution

$$m \, . \, (v^2/l) = T - m \, . \, g \cos\theta,$$

and the energy equation

$$m \, . \, v^2/2 = m \, . \, g \, . \, l \, . \, (\cos\theta - \cos\alpha),$$

in which α specifies the angular amplitude of the oscillations, we deduce the equation

$$T = m \, . \, g \, . \cos\theta + 2 \, . \, m \, . \, g \, . \, (\cos\theta - \cos\alpha)$$
$$= m \, . \, g \, . \, (3 \, . \cos\theta - 2 \, . \cos\alpha).$$

Hence $$T - m \, . \, g = m \, . \, g \, . \{2(1-\cos\alpha) - 3(1-\cos\theta)\}$$
$$= m \, . \, (g/l) \, . \, (2 \, . \, h - 3 \, . \, z),$$

if h, z specify the heights of B, P (or D, R) above C;

and, therefore, $$-m \, . \, g \, . \, h/l \leqslant (T - m \, . \, g) \leqslant 2 \, . \, m \, . \, g \, . \, h/l.$$

The error in taking $m \, . \, g$ for T is therefore a small quantity of the order of h; and therefore a small quantity of the second order relative to the measure of the horizontal amplitude of the oscillations. And the S.H.M.

approximation to the equation of horizontal resolution involves the neglect of a term which is of the third order relative to the terms retained.

Note that $T=m.g$ when $z=2.h/3$, giving a simple specification of the position in which the tension of the moving pendulum is equal to the equilibrium tension.

D. K. PICKEN.

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ANSWER TO QUERY.

[**71. vol. v. p. 330**] Mr. Shovelton's proof (vol. vii. p. 153) may be shortened by using central-difference operators. The theorem itself may be made clearer by splitting it up into two:

(I) If $\phi(x)$ is an odd polynomial in x, and

$$\phi(x+\tfrac{1}{2}) \equiv A_0 + A_1(x, 1) + A_2(x, 2) + A_3(x, 3) + \dots,$$

then

$$A_0 - \tfrac{1}{2}A_1 + (\tfrac{1}{2})^2 A_2 - (\tfrac{1}{2})^3 A_3 + \dots = 0.$$

(II) If $\psi(x)$ is an even polynomial in x, and

$$\psi(x) \equiv B_0 + B_1(x, 1) + B_2(x, 2) + B_3(x, 3) + \dots,$$

then

$$B_0 - \tfrac{1}{2}B_1 + (\tfrac{1}{2})^2 B_2 - (\tfrac{1}{2})^3 B_3 + \dots = \psi(0).$$

To these we may add:

(III) If $\psi(x)$ is an even polynomial in x, and

$$\psi(x+1) \equiv C_0 + C_1(x, 1) + C_2(x, 2) + C_3(x, 3) + \dots,$$

then

$$C_0 - \tfrac{1}{2}C_1 + (\tfrac{1}{2})^2 C_2 - (\tfrac{1}{2})^3 C_3 + \dots = \psi(0).$$

The following properties are involved, the operators being for increment 1 in x:

(i) If $F(x) \equiv P_0 + P_1(x, 1) + P_2(x, 2) + P_3(x, 3) + \dots$, then, as Mr. Shovelton points out, $P_0 = F(0)$, $P_1 = \Delta F(0)$, $P_2 = \Delta^2 F(0)$, ...

(ii) $1 + \tfrac{1}{2}\Delta = \tfrac{1}{2}(E+1) = \tfrac{1}{2}(E^{\frac{1}{2}} + E^{-\frac{1}{2}})E^{\frac{1}{2}} = \mu E^{\frac{1}{2}}$.

(iii) If $\phi(x)$ is an odd polynomial in x, then $f(\delta^2)\phi(0) = 0$; and, in particular, $\mu^{-1}\phi(0) = 0$.

(iv) If $\psi(x)$ is an even polynomial in x, then

$$f(\delta^2)\psi(x) = f(\delta^2)\psi(-x) = f(\delta^2)\tfrac{1}{2}\{\psi(x) + \psi(-x)\};$$

and, in particular,

$$\mu^{-1}\psi(\tfrac{1}{2}) = \mu^{-1}\psi(-\tfrac{1}{2}) = \mu^{-1}\tfrac{1}{2}\{\psi(\tfrac{1}{2}) + \psi(-\tfrac{1}{2})\} = \mu^{-1}\mu\psi(0) = \psi(0).$$

Hence we have the following:

(I) $A_0 - \tfrac{1}{2}A_1 + (\tfrac{1}{2})^2 A_2 - (\tfrac{1}{2})^3 A_3 + \dots$

$$= (1 + \tfrac{1}{2}\Delta)^{-1}\phi(\tfrac{1}{2}) = \mu^{-1}E^{-\frac{1}{2}}\phi(\tfrac{1}{2}) = \mu^{-1}\phi(0) = 0.$$

(II) $B_0 - \tfrac{1}{2}B_1 + (\tfrac{1}{2})^2 B_2 - (\tfrac{1}{2})^3 B_3 + \dots$

$$= (1 + \tfrac{1}{2}\Delta)^{-1}\psi(0) = \mu^{-1}\psi(-\tfrac{1}{2}) = \mu^{-1}\psi(\tfrac{1}{2}) = \mu^{-1}\mu\psi(0) = \psi(0).$$

(III) $C_0 - \tfrac{1}{2}C_1 + (\tfrac{1}{2})^2 C_2 - (\tfrac{1}{2})^3 C_3 + \dots$

$$= (1 + \tfrac{1}{2}\Delta)^{-1}\psi(1) = \mu^{-1}\psi(\tfrac{1}{2}) = \mu^{-1}\psi(-\tfrac{1}{2}) = \mu^{-1}\mu\psi(0) = \psi(0).$$

We can express (I) more generally thus:

(IV) If $\phi(x)$ is an odd polynomial in x, and

$$\phi(x+n) \equiv A_0 + A_1(x, 1) + A_2(x, 2) + A_3(x, 3) + \ldots,$$

then $$A_0 - nA_1 + n(n+\tfrac{1}{2})A_2/2! - n(n+\tfrac{1}{2})(n+1)A_3/3! + \ldots = 0.$$

For this expression is equal to

$$A_0 + \tfrac{1}{2}(-2n, 1)A_1 + (\tfrac{1}{2})^2(-2n, 2)A_2 + (\tfrac{1}{2})^3(-2n, 3)A_3 + \ldots$$
$$= (1+\tfrac{1}{2}\Delta)^{-2n}\phi(n) = \mu^{-2n}E^{-n}\phi(n)$$
$$= \mu^{-2n}\phi(0) = 0.$$

The connecting relation between (I), (II) and (III) is:

(v) If $\psi(x)$ is an even [odd] polynomial in x,

then $\psi(x+1) - \psi(x)$ is an odd [even] polynomial in $x+\frac{1}{2}$.

For it $= \delta\psi(x+\frac{1}{2})$, and δ is an odd polynomial in d/dx. Similarly

(vi) If $\psi(x)$ is an even [odd] polynomial in x,

then $\psi(x+1) + \psi(x)$ is an even [odd] polynomial in $x+\frac{1}{2}$.

W. F. SHEPPARD.

REVIEWS.

A Theory of Time and Space. By ALFRED A. ROBB, M.A., Ph.D. Cambridge. Pp. 16. 6d. net. 1913. (W. Heffer & Sons, Ltd.)

The author hopes to develop in detail the theory of which a short account is given here. The theory in question is an investigation of the relations of time and space in connection with the physical phenomena of optics, and is connected with the theory of relativity. The essential physical considerations of the theory of relativity are attributed to Larmor and Lorentz: Larmor first showed that the electro-magnetic equations could, by a linear substitution, be made to assume the same form when taken with respect to a system moving with uniform velocity as they had when taken with respect to a system "at rest," and similar results were arrived at by Lorentz. This seemed to indicate that, even if such a thing as "absolute rest" did exist, we should not be able in this way to distinguish it from motion with a uniform velocity. It was in order to preserve symmetry that Einstein made the suggestion that events might be simultaneous to one observer, but not simultaneous to another. In Mr. Robb's *Optical Geometry of Motion: A New View of the Theory of Relativity* (Cambridge, 1911), he put forward an outline of a method of treatment, in which he avoided any attempt to identify instants of time at different places. The view was advanced that the axioms of Geometry might be regarded mostly as the formal expression of certain optical facts. Here he gives a short sketch of a treatment of fundamental ideas. By a geometrical interpretation involving more than one dimension, he proves the obvious fact that, with an aggregate which is not linear, if A is neither "before" nor "after" B, A and B are not necessarily identical; and then puts forward the view that the only events which are really simultaneous are events which occur at the same place. Thus, Mr. Robb is of the opinion that, although the set of instants of which any one individual is directly conscious, or the set of instants which a single particle of matter occupies, is a set in linear order, yet the aggregate of all instants does not form a set in linear order. There would seem to be paradoxes implied in the view that time is not uni-dimensional, but it is difficult to see the point of the theory in the short sketch given by the author.

Leçons sur les Singularités des Fonctions analytiques. By PAUL DIENES. Pp. viii + 172. 1913. (Gauthier-Villars, Paris.)

These lectures were given at the University of Budapest, and the volume containing them forms part of the well-known collection of monographs on the theory of functions, edited by M. Borel.

The systematic study of the singularities of analytic functions was begun by Hadamard, and his own results, together with those of Fabry, Leau, Le Roy, and others, were set forth in Hadamard's wonderful little book of 1901 on *La série de Taylor*. The work of Borel and Mittag-Leffler on the representation of analytic functions opened the way for a general theory of singularities, and M. Dienes gives a first sketch of such a theory in the present volume. His point of view is to make no restrictive hypothesis on the coefficients of the development, but to consider more or less particular singularities. He tries to represent the singularities by the nature of the divergence presented by the representation of the function at the point considered. The five chapters are devoted respectively to the researches of Hadamard and the order of a singular point, to a study of the singularities on the circle of convergence, to Borel's method of exponential summation, to the study of singularities by Mittag-Leffler's method, and to the general problem of singularities. The concise account of much of the recent work of Hadamard, Fatou, Fejér, Knopp, Schnee, Marcel Riesz, Borel, Hardy, and P. and V. Dienes makes this book a worthy successor of other volumes in the collection.

PHILIP E. B. JOURDAIN.

The Theory of Structures. By A. MORLEY. Pp. 574, with 4 plates. 1912. (Longmans, London.)

The development and treatment of the Theory of Structures in Professor Morley's latest work compares very favourably with what is to be found in American works on this subject, reviewed in recent numbers of the *Mathematical Gazette*. The great superiority of this treatise is apparent not only in the more strictly mathematical portions, but also in the descriptive parts and the practical details. The book is a great deal more than an advanced text-book for Schools of Engineering, though indeed it is admirably suited for this purpose. Every chapter has a great many examples worked out in detail, and the mathematical results in the text are constantly illustrated by numerical cases. Many results in the more theoretical parts of the work, such as on the Elasticity and Deflection of Beams, are worked by alternative methods : nothing brings home to the student of an enquiring turn of mind the principles of and the relations between the different sections of his subject more than the pleasure of arriving at his previous result by a different route. An indication of the contents of the chapter on Working Stress will give some idea of the thoroughness and completeness of the whole treatise. The mechanical and elastic properties of iron and steel are first discussed; then the importance of ductility as a safeguard against failure, the comparison of steels, containing different percentages of carbon and obtained by different processes, and the effects of temperature; lastly, an interesting historical summary on the effects of Live Loads and Impact Stresses. Wöhler's pioneer research work on this topic (*circa* 1870) was followed by work on similar lines by many other investigators, notably by J. H. Smith and Osborne Reynolds, and recently by Stanton and Bairstow at the National Physical Laboratory. In the experiments conducted by these last investigators the reversals of stress were made at a rate of about 800 per minute, while in Wöhler's original work the frequency was only 60 per minute. Reynolds and Smith, on the other hand, used a reciprocating weight which had a frequency of reversal varying from 1400 to 2400 per minute. Contrary to what one might imagine, the results obtained by Stanton and Bairstow agree much more closely with those of Wöhler (making allowance for the difference in the materials) than they do with those of Reynolds and Smith, the change of frequency from 60 to 800 being less harmful to the material than the change from 800 to 2000. Two of the results of a committee of the American Railway Engineering and Maintenance of Way Association on impact stresses on bridges are quoted by

Professor Morley, and they appear to be of such general interest that they will bear repetition here :

"(1) The chief cause of impact stresses is found in unbalanced driving wheels of locomotives; (2) serious impact stresses arise principally from cumulative vibration resulting from the near approach of the period of rotation of driving wheels at a critical speed to the frequency (*sic*) of vibration of the loaded structure."

Two chapters are devoted to the deflection of the members of indeterminate frames, and the difficult subject of secondary stresses is also alluded to. The practical design of Structures is not lost sight of; full details and working drawings are given for four typical structures : (1) A well-known roof truss, (2) an N girder, and (3) and (4) plate girder bridges. There are a few trifling misprints, chiefly in the mathematical analysis, which have escaped notice in the reading of the proofs. Though none of these are at all serious, it may be worth while mentioning the pages on which they occur—20, 77, 91, 119, 209, and 236. R. M. M.

Problèmes d'Analyse Mathématique. By E. FABRY. Pp. 460. 12 frcs. 1913. (Hermann, Paris.)

Problèmes de Mathématiques Générales (Analyse et Mécanique) données à la Sorbonne et dans les Facultés de Province pendant les dix dernières années. By A. TÉTREL. Pp. 160. 1913. (From the Author, 58 Boulevard Saint-Marcel, Paris, v^e.)

Exercices d'Algèbre, d'Analyse et de Trigonométrie. By P. AUBERT and G. PAPELIER. Vol. I. Pp. 362. 1912. Vol. II. Pp. 360. 1910. 6 frcs. each. (Vuibert et Nony.)

Problèmes de Baccalauréat. By H. VUIBERT. 6th edition. Pp. 527. 5 frcs. n.d. (Vuibert et Nony.)

Esercizi di Analisi Infinitesimale. By G. VIVANTI. Pp. viii + 470. 15 lire. 1913. (Mattei, Pavia.)

Exercices et Compléments de Mathématiques Générales. By H. BOUASSE and E. TURRIÈRE. Faisant Suite au Cours de Mathématiques Générales de H. Bouasse; comprenant, outre l'Étude des Courbes et Transformations usuelles, Les Eléments de la Géométrie du Compas, des Systèmes articulés, du Calcul des Séries, du Calcul des Différences Finies, du Calcul des Probabilités, du Calcul Vectoriel. Pp. xxv + 500. 18 frcs. net. 1913. (Delagrave, Paris.)

These collections of problems, for the most part worked out, will certainly be of use to the private student who has to rely on his native instinct as to whether or not his own solution contains the last word, and who is unaware of shorter or more elegant methods of solution than those to which he is naturally drawn. A large number of the problems are new, and we cannot but think that those who have to set examination questions might well pass an idle moment or so in turning over their pages, and in so happy a hunting-ground get sundry hints, and perchance at the same time learn what to avoid. Some will be found of use to the scholarship candidate for revision purposes, and from many the teacher will get wrinkles. Particularly is this the case with the remarkable volume by MM. Bouasse and Turrière. It appears that the purpose of this notice is attained if we confine it in the main to indications of the ground that is covered.

M. Fabry has classified 279 questions set at various examinations during the last decade and provided them with solutions. The latter are sometimes given in detail, and at others are merely sketched, so that there is plenty of room left for individual effort on the part of the student. In many cases the questions contain enough material to make half a dozen problems such as are set in our three hours' papers. The headings of the twelve sections indicate the ground covered : areas, multiple integrals, surfaces and volumes, analytical functions and curvilinear integrals, differential equations, plane curves, skew curves and surfaces, asymptotic lines and lines of curvature, ruled surfaces, partial differential equations with geometrical applications, total differential equations, and elliptic functions. Of

the solutions as far as we have tested them we can speak highly. We may add that the questions are printed separately from the solutions. For school or other purposes it would have been better had the questions been published under a separate cover.

The two volumes of Exercises in Algebra, Analysis and Trigonometry by MM. Aubert and Papelier are intended for the use of students of the first and second year "de Mathématiques Spéciales" respectively. The first 72 pages of Volume I. contain 138 questions, for the most part with hints or completely worked out, dealing with algebraical manipulations, determinants and their application to the solution of linear equations, and convergency and divergency of series. Elementary theory of equations is given 50 pp., and trigonometrical formulae, series, roots of imaginary numbers, and solution of triangles 45 pp. The sections on differential calculus take us through the introductory matter, variation of functions, expansion in series with applications, and cover some 90 pp. Two chapters are given to integrals and their application to areas, lengths and volumes. The year's course is closed with about 40 pp. on easy differential equations. The second year course begins with 48 pp. on combinatory analysis, the binomial, and a more advanced treatment of determinants. Types of the questions now dealt with in the differential course are: "Find the most general functions $f(x)$ and $\phi(x)$ such that whatever be the values of x and y the expression $V^2=(x^2-y^2)[f(x+y)+\phi(x-y)]$ may be written in the form $F(x)-\phi(y)$. Find the functions $F(x)$ and $\phi(y)$." "Given $x^y=y^x$. Find the value of dy/dx. In this value put $y=x$ and explain the result. Construct the curve. Find the tangent at the point $x=y=e$." The sections on expansion of series are carried to a point that may be estimated from the question: "On a plane curve C take a point O presenting no singularity, and refer the curve to the tangent and normal at O as axes of x and y. Suppose that there is a variable point M on the curve, the coordinates of which may be expressed by series arranged in positive and integral powers of the length s of the arc OM. Express the coefficients of the first terms of the series up to s^6 as functions of the values R, R_1, R_2, R_3 of the radius of curvature R of the curve C at O and its derivatives with respect to s. Find the coordinates of the mid point P of a chord MM' parallel to the tangent Ox and of infinitesimal length." As an instance of the kind of problem worked out in the chapter on geometrical applications of the calculus, we may take: "A curve S traced on a cone of revolution meets the generators of the cone at a constant angle. Find the parametric equations of the curve S; the evolute S' of S; show that the tangents to S and S' at corresponding points are perpendicular; and show that S is on a cone of revolution and cuts its generators at a constant angle." The course on differential equations is carried on to partial differential equations. The volume concludes with a chapter on algebraical equations containing questions the difficulty of which may be gauged from this: "A polynomial $f(x)$ of degree n satisfies the identity $n.f(x)=(x-a)f'(x)+b.f''(x)$. Find the coefficients of $f(x)$ arranged in powers of $x-a$. Discuss the conditions that the roots may be real. Show that if b_0 is the absolute value of b, the roots of $f(x)$ lie between $a\pm\sqrt{n(n-1)b_0/2}$."

The popularity of M. Vuibert's *Problèmes de Baccalauréat* is shown by the fact that this collection of over 500 pages of problems and solutions has reached a sixth edition. Here the problems set, and in most cases solved, are of a more elementary order. Algebra is carried as far as problems and simultaneous equations of the second degree, maxima and minima, progressions, logarithms, and annuities. The Geometry is more complete, covering the contents of the first six books of Euclid, calculation of volumes, etc., and questions of such difficulty as the following: "A is the pole of BC the chord of a circle centre O. Express in terms of the radius R and of the projection BE of the arc CFB on OB the volumes of the bodies generated by (i) the segment $BCFE$, (ii) the triangle BOC; and (iii) the curvilinear triangle $BFCA$ as the three figures revolve round OB. Discuss the particular case in which $BE=BO$, *i.e.* when $BOC=90°$." A few easy questions are set on the ellipse and parabola. The questions in Trigonometry are both theoretical and practical. The problems in Descriptive Geometry occupy 15 pages, and their solutions of those selected with their diagrams fill 14 pages. The Mechanics is of about the standard of the Senior Locals or a very easy Scholarship paper. The last section is given to Cosmography. Here are samples: "Two astronomers observe at the same moment a star whose zenithal distance is $Z=30°$ on the horizon of the first

observer and $Z=45^\circ$ on that of the second. The sum of the two actual parallaxes of the star with reference to the two observers is $70'$. Find the distance of the star from the centre of the earth in terms of the earth's radius." Or again: "Mercury's period of revolution round the Sun is 88 mean days. Find the semi-major axis of the planet's orbit in terms of that of the terrestrial orbit." Teachers will be interested in the methodic fashion in which logarithmic work in the solution of triangles is set out.

Prof. Vivanti's *Esercizi di Analisi Infinitesimale* are two-thirds of them new. Among those borrowed from well-known sources we note a few from Boole's Differential Equations and Forsyth's treatise. Out of the 470 pages, 10 are given to elementary differentiation, 20 to limits and indeterminate expressions, 40 to applications of Taylor's Theorem and to maxima and minima. To feats of integration are devoted 70 pages, followed by applications of the calculus to plane curves, about forty curves being traced, their singular points determined, and lengths of arcs and areas calculated. On envelopes, curvature, and evolutes twenty-nine questions are worked out, and then we have 38 pages on osculating planes, circles, and spheres, and on flexion and torsion. The remaining 180 pages or so are given to differential equations. The total number of problems worked out is close on 600. The solutions are for the most part full and clear, the diagrams are well drawn, and the text is widely spaced and beautifully printed on excellent paper. The book is a pleasure to read.

M. Tétrel's collection consists of 181 fully worked-out questions under the headings: Series, Expansions in Series, Differential Equations, Kinematics, Dynamics, and General Problems. Here is one from the last group: "Find the locus of points on $x=(a+z)\cos t$, $y=(a-z)\sin t$, at which the tangent plane is parallel to Oz (a is a constant and t a variable parameter). A point moves on the surface under a force represented at each moment by a vector directed along the generator through the point, and having for its projection on the axis of z the expression $z+f(t)$, where $f(t)$ is a function of the variable t. Find the function $f(t)$ if the work of the force for any displacement of the point on the surface depends only on the origin and the final position of the point. Determine the work done." Or: "Find the locus of the traces on the plane xOy of tangents to the curve $x=z^2$, $y=\frac{2}{3}z^3$. Find the law of a heavy point moving on this curve without friction. The axis of z is supposed vertical and the point is initially at rest at the origin." Among the applications of analysis we find questions such as this: "Construct the curve $\rho=a\tan^{\frac{1}{2}}\frac{\omega}{2}$. (C), in which a is a given positive length. Find the Cartesian equation to the curve and the coordinates of a point upon it in rational functions of a parameter. If OA be an arc of the curve above the axis of x and to the right of Oy, and T be the point of intersection with Ox of the tangent at A, find the area of the part of the plane between OA, the tangent AT and the axis Ox. Construct the circle of curvature at A. Find the orthogonal trajectories of the curves as a varies. Construct one of these curves. Calculate for $a=1$ the coordinates of the point furthest to the right on the curve C." This is taken from the first examination for the Lille electrical engineering diploma. A similar volume of solutions to the questions in electricity at these examinations has been published by the author.

The 500 pages of *Exercices et Compléments* issued by MM. Bouasse and Turrière is a sequel to M. Bouasse's *Cours de Mathématiques Générales*. Here we are on quite a different plane. It is intended in the main for the future engineer and physicist, and will attract all those who do not mind any effort of intellectual acquisition as long as they can clearly see that it is closely related to the occupation by which they are to gain a livelihood. It is an authoritative pronouncement of the modern French view as to the position of mathematics in a general scientific culture. Its essential characteristic is that, although the purview is to be wide the grasp of the subject is to be real. The one thing that must be insisted upon is that what is to be done is to be done thoroughly. This is secured in two ways by the course laid down in this volume. The student is led to recognise the general or particular value of the work on which he is engaged in the realities of daily life, *e.g.* if he is set to trace a certain cubic he hears something of Van der Waals and the saturation curve. But the method adopted is one which goes far to remove the reproach that a course of such a kind tends to make the student

merely a craftsman skilled in the use of his tools or a "mathematical acrobat." The exercises are devised also to develop the faculty of abstract reasoning. Tait has warned us that a text-book should not be too easy reading. It is one thing to ascend the dreary monotony of a marble staircase. But it is a better thing to reach the height at which we aim by our own carefully cut steps. What the authors do is this. They accompany the climber in his ascent. At first they have to tell him how and where to place his feet, where and how to cut the next steps. Presently this may for the most part be left to his own judgment, and they intervene only in cases in which he may be led to a *cul de sac* or to a cliff face which perhaps they themselves have once tried in vain to scale. "This is a course in which a considerable part of the work is left to the reader. We show him the road and tell him what he must aim at doing. We lay out the various stages for him, but he must supply the gaps in our work." The preface of M. Bouasse must be read to be appreciated, and it is well worth reading. Perhaps an idea of the authors' method will be obtained if we transcribe a complete section.

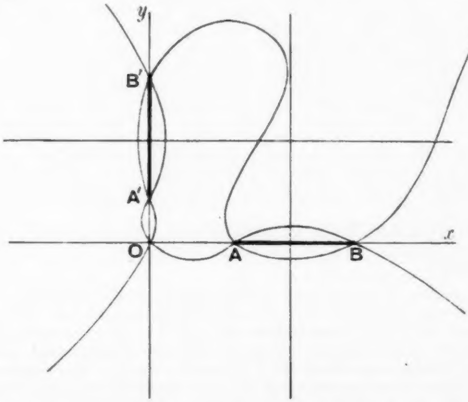
We select one dealing with a problem that has more than once been discussed in the *Gazette*:—"140. Locus of points at which two segments subtend equal or supplementary angles.

"(i) In the preceding exercise we found the point at which two segments subtend equal angles and also satisfy a further condition. Let us now consider the general problem of the locus of points at which two segments always subtend equal angles. An elementary construction gives us as many points as we wish. Let $l=AB$, $l'=A'B'$ be the lengths of AB, $A'B'$ respectively, and let these lines intersect in O. Draw through their extremities circles of radii kl, kl', where k is any factor. The intersections of these circles lie on the locus.

"But the reader will find that it is difficult to trace the curve, because it is generally composed of two cubics. These cubics are called circular cubics, because the terms of the third degree are of the form $(ax+by)(x^2+y^2)$. And he will find that it is very easy to make mistakes in joining up the points obtained. Let us begin with the study of particular cases.

"Show that the points O, A, B, A', B' belong to the locus. Find the direction of the asymptotes.

"(ii) Suppose $AB=A'B'$ and that they lie respectively on the rectangular axes of coordinates, origin at O. And to simplify the work take $OA=OA'=1$; $OB=OB'=2$.



"The analytical solution is then very simple, reducing to finding the intersections of the circles $x^2-3x+2+y^2+ay=0$; $y^2-3y+2+x^2\pm ax=0$.

"Taking the upper sign, the locus is a cubic which decomposes into the straight line $x=y$, and the circle $x^2+y^2-3(x+y)+2=0$.

"In the other case the locus is a cubic. Find its asymptote. Trace the two curves. Explain how it is that there are two signs.

"(iii) Suppose the segments to be equal and to lie on two parallel straight lines. Take a parallel to the two segments as the axis of x, and as origin the point to which they are symmetrical, Consider the case in which Oy bisects the segments. Trace the curve. Show how by the principle of continuity it may be derived from case (ii). Treat in the same way the case in which Oy does not bisect the segments. Show how the system composed of the straight line and circle found above gives a cubic properly so-called (like an S with two asymptotic branches). As a limiting case suppose the segments on the same straight line: in this case we take them of unequal length.

"(iv) Suppose the equal segments to be on two perpendicular lines, but no longer at the same distance from the origin. The figure represents the curve obtained. Determine the direction and position of the asymptotes. From this curve exhibit the preceding curves as particular cases. Taking the segments as equal simplifies materially the work and does not affect the generality of the solutions. Finally remove the restriction $B\hat{O}B'=90°$. The shape of the curves is not thereby altered."

From the table of contents we take the following: Tracing of curves; curves constructed by the aid of simple curves; the triangle and quadrilateral; articulated polygons; homographic transformations; inversion; transformations by means of the square and the jointed rhombus; transformations by means of imaginaries; displacement of an invariable figure; displacement of a moving plane defined by the displacement of a line, equipotential lines and surfaces; lines of force; Roberval's transformation; reciprocal polars; problems on differential equations and integrals; volumes and areas; finite differences; recurrence; approximations; probabilities. The diagrams are beautifully drawn, and the book is printed in large type and on excellent paper. The price (18 frcs.) is net—we notice this, as it is unusual to find a French price stated as net.

A Text-Book of Mathematics and Mechanics. By C. A. A. Capito. Pp. xv+398. 12s. 6d. net. 1913. (Griffin & Co.)

Mr. Capito's *Text-book of Mathematics and Mechanics* is "specially arranged for the use of students qualifying for Science and Technical Examinations." It begins with Analytical Geometry, runs through the Calculus, and closes with Mechanics and Hydrostatics—all within some 400 well printed and clearly spaced pages. A knowledge of elementary mechanics is assumed, with a sound complement of Geometry, Algebra, and Plane Trigonometry. A feature of the book is the large number (250) of questions taken from the Qualifying Examination of the Mechanical Science Tripos, that of the Associate Membership of the Institution of Civil Engineers, etc., and worked out in full. The expository power of the author is considerable, and the engineering student who has carefully gone through the book will have nothing to unlearn. We fear, however, that the price of this volume (12s. 6d. net) will tell heavily against its sales.

Cours d'Analyse de l'École Polytechnique. By C. Jordan. 3rd edition, revised and corrected. Vol. ii. Calcul Integral. Pp. 705. 20 frcs. 1913. (Gauthier-Villars.)

The third edition of the volume of this course dealing with the Differential Calculus appeared about four years ago. It is perhaps as well to remind those who wish to consult the second volume that the definition and fundamental properties of the definite integrals, with the fundamental notions of the infinitesimal Calculus, are dealt with in vol. i. In the new edition of vol ii. the work of Schmidt on potential theory has enabled the author to recast the chapter on that subject and to bring it up to date. Some twenty pages are devoted to its applications in electricity and magnetism. There is also a new chapter on vector fields, introducing the student to the theorems and formulae connected with the names of Ostrogradsky, Stokes, Green, Dirichlet, and Liouville.

The Development of Mathematics in China and Japan. By Y. MIKAMI. Pp. viii+347. 18 m. 1913. (Teubner.)

"That English is already the world language, what proof could be more striking than its use in a history of Chinese mathematics written in Japan, revised in America, published in Europe?" Thus begins the characteristic preface supplied by the reviser, Prof. G. B. Halsted, to this substantial contribution towards a history of oriental mathematics. In his introductory note the author takes the opportunity of laying various spectres which have many times daunted those who have made their little incursions into this department of the history of our science. The lack of uniformity in the spelling of proper names, due largely to misinterpretation of ideograms, has been a real difficulty and the cause of many errors. Even the Japanese themselves differ as to the reading of the individual names. The Yasushima of R. Fujisawa is the same person as the Ajima of D. Kikuchi. Seki Kowa should be Seki Takahazu. Strangely enough, the family traditions in a country of ancestor worship cannot be relied upon for the correct renderings of notable forefathers. The author makes no attempt to undertake the futile task of making a comparison between the mathematics of the East and of the West—a task as futile as the endeavour to draw a parallel between the Pythagoreans and any modern school of mathematicians.

One sees from the outset that the use of some sort of abacus gave the impulse to early mathematical thought. The calculating pieces were of bamboo or wood, and it is most interesting to learn that the red pieces represented positive and the black represented negative numbers. The oldest Chinese mathematical treatise extant discloses the fact that twelve centuries before the Christian era the Chinese were aware of the theorem of Pythagoras—and Pythagoras flourished six centuries later. The close relation between religious rites and the calendar will be sufficient to account for the prominence given in all the early treatises to matters connected with time and the heavenly bodies. From the earliest historical period the Chinese had taken 3 for the value of π and the year as $365\frac{1}{4}$ days, had divided the circumference of the circle into $365\frac{1}{4}$ degrees, and in the third century B.C. they could divide an integer by a fraction. Unfortunately, by an edict of "the mighty and despotic emperor, Shih Hoang-ti in the year 213 B.C. burned all books and buried all scholars." But in the course of time even Shih Hoang-ti and his empire met the fate of Ozymandias, old writings were recovered, and a renaissance began. In due course appeared the *Chiu-chang Suan-shu* or *Arithmetic in Nine Sections,* the first really arithmetical treatise. How much of it was due to the genius of Ching and how much to Chang it is at this distance of time idle to debate. Suffice it to say that in 150 B.C. the value of π was still taken as 3, that fractions could be simplified, added, subtracted, multiplied, and divided, but that, in spite of the secular use of the abacus, the idea of decimals had not yet suggested itself. This led to difficulties in the extraction of roots. "When evolving, if we don't come to a conclusion, the number cannot be evolved indefinitely. In such a case, divide the remainder by the evolved root, and take the result to the fractional part of the root." In mensuration correct values are already being attained. Equations are solved by methods not essentially differing from our own, even as far as simultaneous equations of the first degree with three unknowns. The next arithmetical classic appears to be that associated with the name of Sun-Tsu, perhaps of the sixth century B.C. As a practical instance of the alliance by this time existing between theory and practice, we may give one of the questions it contains, and anticipating the clamour we foresee may be raised for the answer, we also give the solution: "A lady of 29 years of age is 'expecting' in the ninth month of the year. Will her child be a son or a daughter?" To Sun-Tsu the problem presents no difficulties. "Take 49; (why 49 is not quite clear; is it a misprint for 29? or is it the square of the number of the planets as known to the ancients, plus the moon and the earth? or is it the square of seven as a sacred number? or again, is 20 added to the 29?); add the month of the birth; subtract 1, 2, 3 for the heaven, the earth, and the man respectively, 4 for the seasons, 5 for the elements, 6 for the six laws, 7 for the seven stars, 8 for the eight winds, and 9 for the nine

provinces. If the remainder be odd, the child will be a son; if even, a daughter." Here, too, we find the first problem in indeterminate analysis. But let us leave what Chou Kong calls "the mighty art of numbers," and come to the circle-squarers. The author's English is extremely creditable, and as a rule it is quite easy to make out what he means. But he is occcasionally obscure, and on one occasion at least he conveys an impression opposite to what is intended. For example, he reminds us that up to the twelfth century B.C. the value of π was taken as 3, "and ever subsequently." Five lines further down we are told that "the Chinese did not in any way remain always satisfied only with this rough value of π." But we can forgive him a good deal for the delicious quaintnesses of style which now and again occur, and which no doubt are the counterparts of what we ourselves would perpetrate if we attempted a similar task. Long before Aristophanes wrote *The Birds* the Chinese had their cyclometrists. By 139 A.D. Chang adopted $\sqrt{10}$ (Wang Fan's 3·1555... was closer), and then came the serious investigations of Liu Hui. He evaluates sides and areas up to the 192-gon, and finds 3·14 extended later by Wu to 3·1432+, and by Tsu Ch'ung-chih, who 200 years after, by a different method reached a value lying between 3·1415927 and 3·1415926, giving the fractional equivalents 355/113 and 22/7. "The Chinese had therefore been possessed of this . . . over a millennium earlier than Europe. We are on this account strongly urged to express a desire that it should henceforth be called by the name of Tsu Ch'ung-chih's fractional value for π."

Space forbids further notice of this most interesting volume, but we hope we have said enough about it to induce our readers to avail themselves of any opportunity they may have of perusing its pages. The author, in collaboration with Dr. D. E. Smith, is engaged at present on a volume devoted to Japanese mathematics, to which we look forward with pleasurable anticipations.

An Introduction to the Mathematical Theory of Attraction. By F. A. Tarleton. Vol. II. Pp. xi+207. 6s. 1913. (Longmans.)

The first volume of Dr. Tarleton's treatise appeared some fourteen years ago. In its preface he announced his intention of completing the book by the addition of chapters on Spherical Harmonics, Conjugate Functions, and the Theory of Magnetism for bodies of finite dimensions. This promise he has now discharged in part. The second volume contains chapters on Spherical and Ellipsoidal Harmonics, on Magnetised Bodies, Electric Currents and Dielectrics. But the proposed chapter on Conjugate Functions does not appear, as riper experience has convinced the author that to the student of to-day an account of the electro-magnetic theory of light would be at once more interesting and more instructive. "Of the more recent developments of the electro-magnetic theory of light I have not attempted to give any account. So far as I can judge, some of these rest on insecure foundations." According to his temperament and advantages the student will be relieved or dismayed by the entire absence of the usual selection of examples for solution.

Cours d'Analyse Infinitésimale. 1ère Partie. **Calcul Différentiel et Calcul Intégral. Méthode simple pour apprendre ces branches des mathématiques supérieures.** By A. J. M. Crefcœur. 2nd edition, revised and enlarged. Pp. 358. 1912. (Beranger, rue des Saints-Pères 15, Paris.)

This course is intended for those who wish to make mathematics part of their general culture, and would be found especially useful by those who are not skilful in algebraical manipulation. We judge this to be the case from the number of steps which are taken in reducing not particularly complicated expressions to simpler forms. Taken as a whole, it may be said that it is a useful introduction to the calculus. Fifty pages are given to differential equations. We can congratulate the author neither on his printer nor on the reader of his proofs. The pages are difficult to read, and the formulae are indifferently set out and ill-spaced. There is nothing to catch the eye, and thus to aid the memory. From almost any page we might cull a word misspelled, an inverted or a capital letter in the middle of a word, a broken

letter, two words run together, or some other instance of bad proof-reading, editing, or printing. Curiously enough, what we might call the purely mathematical portion of the book—the formulae, equations, etc.—is as a rule quite correct. If this is the second edition, we can but shudder to think what the first must have been like.

Eloges Académiques et Discours. By GASTON DARBOUX. Edited by the Committee of M. Gaston Darboux's Scientific Jubilee. Pp. 525. 5 frcs. 1912. (Hermann, Paris.)

The genesis of the above volume is honourable to Professor Darboux. In 1911 he completed his fiftieth year of service as a teacher, and his twenty-fifth as a member of the *Académie des Sciences*—for ten years he had been its permanent secretary. It seemed to his pupils, admirers, and friends that it would be appropriate to celebrate this "university golden wedding" and "academic silver wedding" by presenting M. Darboux with a medal. A committee was formed comprising the names of mathematicians in all quarters of the world. The funds subscribed permitted them to present to each subscriber a copy of this volume, containing not only Darboux's speeches and official addresses, but also those delivered on the occasion of the Jubilee, with the names of the subscribers. When a member is removed by death it is part of the duties of the permanent secretaries of the *Académie des Sciences* to commemorate by an oration his life and work. The practice was instituted by Fontenelle. The expositor of the philosophy of Descartes and the famous author of *Entretiens sur la Pluralité des Mondes* took part in the great quarrel of the seventeenth century between the ancients and the moderns. By the efforts of those who took the side of the ancients—in particular, Racine and Boileau—his application was four times rejected. But the moderns prevailed in the end, and not only was he elected to both the *Académie des Inscriptions* and the *Académie des Sciences*, but for over forty years he was permanent secretary to the latter body. He delivered some seventy of these orations, including one on his uncle Pierre Corneille. Among the more famous of his successors in this office was Condorcet, who, disguised as a carpenter, was betrayed to the vengeance of the Revolution by the delicacy of his hands, and by not knowing how many eggs should go to an omelette. He published the *éloges* he delivered between 1666 and 1699. If the orations of Condorcet were graceful and touched with the eloquence one would expect from his emotional nature, those of Arago were simple and lucid. A selection was published in English in 1857 by Admiral Smyth and others, Bailly, Herschel, Laplace, Fourier, Carnot, Fresnel, Thomas Young, and James Watt being among those chosen. The immediate predecessors of M. Darboux were Berthelot, Lapparent, Van Tieghem, and J. Bertrand. If we remember rightly, it used to be the practice of the Smithsonian Institute of earlier days to translate the best of the orations. We are not aware that any translations have been made into English during the last forty years.

Darboux's first effort was, of course, in praise of his immediate predecessor, J. Bertrand, to whom he was deeply attached. Bertrand, like Darboux, was permitted to celebrate his "university golden wedding," and, like him, received from his pupils and friends a medal in honour of the occasion—an honour which, as he remarked, had been rendered neither to Lamé nor to Cauchy. His last years were the happier for the fact that his eldest brother was elected to the *Académie des Inscriptions*, and his son Marcel, with his nephews Emile Picard and Paul Appell, to the *Académie des Sciences*. We need not dwell upon the next in order—François Perrier, noted for his service to French Geodesy. He is followed by Hermite. Richard, the professor at the *Collège Louis-le-Grand*, who fifteen years before had taught Evariste Galois, prophesied to Hermite's father that his son would be a little Lagrange. *En passant*, it may be noted that in voice and appearance Hermite bore an astonishing resemblance to Galois. But, like other teachers of famous men, Richard complained bitterly of his pupil's disinclination to pursue the ordinary routine in preparation for his examinations. As Pollock said of Clifford, he omitted most of the things he ought to have read, and read everything he ought not to have read. It was at the age of eighteen that the

furie mathématique developed with extraordinary intensity and rapidity. He spent his time in the library reading the works of great classics like Euler instead of preparing himself for the fateful interrogatory in Geometry and Mechanics. He won his first *Concours Général*—the subject being the theory of elimination. Just at this time the quality of the questions in the *Concours* was sharply criticised by Terquem, who observed that ever since the control of mathematical teaching had passed at his death from the hands of illustrious Poisson, things were going from bad to worse as far as the *Concours* was concerned—*nous convergeons vers l'antique Byzance.* The reference is to the experience of Baron de Tott of the Embassy to the Porte, who was asked by a very ignorant vizier to set a test question in mathematics to the *Collège de Constantinople*, in which he was deeply interested, and of which, as will be seen, he had an unduly high opinion. The Baron, diplomatically anxious not to wound the feelings of a minister whose good offices he might one day require, after due deliberation asked for a proof that the sum of the angles of a triangle is two right angles. After the lapse of a few days the answer was returned that the property is true in the case of an equilateral triangle. Of Hermite's precocity Darboux remarks that is is impossible to read without a feeling of wonder the four letters to Jacobi in which Hermite announced his discoveries in higher Arithmetic.

He may be compared to Galois, who made his immortal discoveries at the age of twenty; or to Gauss, who at the same age wrote his *Disquisitiones Arithmeticae*; or to Lagrange, who at twenty-three had laid the foundations of all his future work. In the case of Hermite these letters surpass anything he had written till then, both in the extent of the problems he attacked and the nature and fertility of the principles employed. They contain the germ not only of his principal discoveries, but also of those the path to which he prepared for his successors. In the eyes of all competent judges, from the moment Hermite wrote these letters, Hermite must be ranked among the great mathematicians.

Hermite married the sister of the two Bertrands, and his younger daughter became the wife of M. Emile Picard. Before his marriage Hermite lived in the same house as Bernouf, with whom he became intimate. Under the inspiration of the great philologist he studied ancient Persian and Sanscrit, being attracted to languages by the mechanism of grammar, finding in it as much pleasure as in an algebraical transformation. Like so many mathematicians, he was extremely fond of music, and could easily pick out on the piano a *motif* which he had once heard. But wild horses would not drag him to see a painting or a piece of sculpture. From this branch of art he had a positive aversion.

Was Hermite a great teacher? Let some of his pupils answer. Those who heard him, says Picard, will never forget his incomparable teaching. His lectures were like wonderful *causeries*. They were delivered in a serious tone, the level of which would now and again be raised in a moment of enthusiasm. Then would suddenly be revealed vast horizons, and by the side of the Science of to-day would suddenly appear the Science of to-morrow. No teacher was ever less didactic or more vivid. Or listen to M. Painlevé. He assures us that those who had the good fortune to be his pupils will never forget the almost religious accent of his teaching, or the thrill of beauty or mystery which would pass over his audience as he revealed some new discovery or drew the veil from the unknown. As a teacher he was incomparable. He focussed curiosity and attention upon new and important problems. It was his crowning merit that he communicated to others his own devotion and respect for ideal truths. On that never-to-be-forgotten day of his Jubilee he spoke in noble terms of the intimate and secret correlation that exists between the sense of justice and of duty, and the knowledge of the absolute truths of our science. It was at the Sorbonne that Borel attended Hermite's course, and, as he tells us, heard him expound so vividly and with such devotion and respect the noble truths of Analysis. He was like a High-Priest of the God of Number unveiling to us awful and sacred mysteries. The driest of questions, calculations apparently of the most unpromising nature, became transfigured in his hands. He seemed to have an intuition of their secret beauties.

Many have had the power to make their students understand and admire mathematics; none but he possessed the secret of making his audience feel their subtle attraction.

And what lesson has he for the teacher? "I confess," he wrote to Stieltjes, "that I sometimes give way to indolence (he was then seventy-two), and do not give sufficient preparation to my Sorbonne lectures. But in the watches of the night my conscience pricks me, and I have no peace until I light the candle and get up and finish what I have left undone." So much for duty. He advised wide reading, holding that even the most elementary teaching may profit from the study of the works of genius when they directly touch its object. "I hope," he wrote, "that for the sake of the student more attention will be paid to the simple and the beautiful, and less to that extreme rigour which is so fashionable nowadays, which is so unattractive, so often fatiguing, and so profitless to the beginner, who naturally cannot appreciate its interest and value." He was himself notable for the attention he paid to the historical side in his teaching, and for the efforts he made to widen the horizon of his pupils. But we must draw to a close. Other notices are those of Abbadie, of General Meusnier (Professor Archibald has searched in vain for the reason why British writers always misspell the name of this author of a famous theorem), Berthelot, Pasteur, and others. A rich harvest awaits the readers of this collection.

CORRESPONDENCE.

To the Editor of the *Mathematical Gazette*.

Sir,—It has been pointed out to me that the introduction to the Report of the Public Schools Special Committee, published in the July number of the *Gazette*, is somewhat misleading in respect of the statement that it is "approved by the General Committee of the Mathematical Association." The exact position is that the Report was "approved *for publication* by the General Committee, etc." The General Committee does not necessarily "approve" the Report in the formal sense of the term.—Yours faithfully,

P. Abbott,
Hon. Sec. of the General Committee.

NOTICE.

Prof. Em. Mougin, President de l'Association Amicale des Lycées de Roanne (France), will be pleased to forward to any member of our Association a specimen copy of:

1. New Tables of Logarithms with 5 decimals (English edition): Logs of Numbers from 1 to 10,000; Logs of Sin, Cos, Tans, Cotans from 0 to 100 grades; Logs of Sin, Cos, Tans, Cotans from 0° to 90°; Natural Sines (0 to 100 grades); Natural Sines (0° to 90°); Supplement and Use of the Tables.

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THE LIBRARY.

THE Librarian acknowledges with thanks the gift of *Theories of Parallelism*, by Rev. W. B. Frankland, and the *Geometry of Thales*, by P. J. Harding, Esq.

The Library has now a home in the rooms of the Teachers' Guild, 74 Gower Street, W.C. A catalogue has been issued to members containing the list of books, etc., belonging to the Association and the regulations under which they may be inspected or borrowed.

The Librarian will gladly receive and acknowledge in the *Gazette* any donation of ancient or modern works on mathematical subjects.

Wanted by purchase or exchange :

1 or 2 copies of *Gazette* No. 2 (very important).
2 or 3 copies of Annual Report No. 11 (very important).
1 or 2 „ „ Nos. 10, 12 (very important).
1 copy „ Nos. 1, 2.

BOOKS, ETC., RECEIVED.

The Modern Geometry of the Triangle. By W. GALLATLY. 2nd edition. Pp. vii + 126. 2s. 6d. net. 1913. (Hodgson, 89 Farringdon St., E.C.)

Practical Mathematics for Students attending Evening and Day Technical Classes. By N. W. M'LACHLAN. Pp. viii + 184. 2s. 6d. net. 1913. (Longmans, Green.)

Essays on Mathematical Education. By G. ST. L. CARSON. Pp. iv + 139. 3s. net. 1913. (Ginn & Co.)

A General Course of Pure Mathematics from Indices to Solid Analytical Geometry. By A. L. BOWLEY. Pp. xii + 272. 7s. 6d. net. 1913. (Clarendon Press.)

Annals of Mathematics. Edited by O. STONE and others. Sept. 1913. I. Vol. 15. 2nd Series. 2$ per vol. (Princeton, N.J.)

Singular Point Transformations in two Complex Variables. G. R. CLEMENTS. *On the Projective Differential Geometry of Plane Anharmonic Curves.* R. W. REAVES. *On the Rank of a Symmetrical Matrix.* L. E. DICKSON. *Note on the Rank of a Symmetrical Matrix.* J. H. M. WEDDERBURN. *On the Numerical Factors of the Arithmetic Forms* $\alpha^n \pm \beta^n$. R. D. CARMICHAEL.

Nouvelles Annales; Il Periodico matematico, e Supplemento; Il Pitagora; L'Enseignement Mathématique; Revue de Mathématiques Spéciales; Intermédiaire des Mathématiciens; Gazeta Matematica. June-Sept. 1913.

The School Algebra. Matriculation Edition. By A. G. CRACKNELL. Pp. viii + 420 + lxv. 4s. 6d. 1913. (Univ. Tutorial Press.)

Vectorial Mechanics. By L. SILBERSTEIN. Pp. viii + 197. 7s. 6d. net. 1913. (Macmillan.)

The Calculus and its Applications. By R. G. BLAINE. Second impression. Pp. ix + 321. 4s. 6d. net. 1911. (Constable.)

A Text Book of Thermodynamics (with special reference to Chemistry). By J. R. PARTINGTON. Pp. viii + 544. 14s. net. 1913. (Constable.)

Hydraulics and its Applications. By A. H. GIBSON. New edition, revised and enlarged. Pp. xvi + 813. 15s. net. 1912. (Constable.)

The Theory and Practice of Mechanics. By S. E. SLOCUM. Pp. xlii + 442. 15s. net. 1913. (Constable.)

Essai de Linéometrie. By J. SER. Part I. Pp. iv + 80. 2 fr. 25. 1913. (Gauthier-Villars.)

Analyse Vectorielle Générale. II. *Applications à la Mécanique et à la Physique.* By C. BURALI-FORTI and R. MARCOLONGO. Pp. xii + 144. 5 frcs. 1913. (Bowes & Bowes, Cambridge.)

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